POPULATION POLICY AND INDIVIDUAL CHOICE: A THEORETICAL INVESTIGATION

Marc Nerlove, Assaf Razin, and Efraim Sadka
The International Food Policy Research Institute was established in 1975 to identify and analyze alternative national and international strategies and policies for meeting food needs in the world, with particular emphasis on low-income countries and on the poorer groups in those countries. While the research effort is geared to the precise objective of contributing to the reduction of hunger and malnutrition, the factors involved are many and wide-ranging, requiring analysis of underlying processes and extending beyond a narrowly defined food sector. The Institute’s research program reflects worldwide interaction with policymakers, administrators, and others concerned with increasing food production and with improving the equity of its distribution. Research results are published and distributed to officials and others concerned with national and international food and agricultural policy.

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Population growth is one of the more important forces affecting both the supply and demand for food in developing countries. It is a critical determinant of the well-being of the lowest income groups—as discussed at length in an IFPRI-Johns Hopkins Press book, *Agricultural Change and Rural Poverty: Variations on a Theme* by Dharm Narain, edited by John W. Mellor and Gunvant M. Desai. Hence population is an important concern in the context of IFPRI's mandate.

Marc Nerlove, whose major effort at IFPRI deals with factors affecting the rate of diffusion of new agricultural technology, joins with Assaf Razin and Efraim Sadka to present us with a tightly reasoned theoretical analysis of population policy and individual choice. Increasingly, as the national research systems of developing countries improve their capacity in applied research and draw in more basic research, institutions like IFPRI will have to move "upstream" in their research, providing a more detailed conceptual framework and additional methodological advances. This work is a tentative step in that direction.

The basic concept of the work is built on the assumption that parents care about their children—an obvious assumption but not one that dominates the theoretical literature. The authors build from this assumption a solid theoretical structure that leads to important conclusions for policy.

Although the conclusions drawn from the theoretical analysis are important, the study also indirectly corroborates how important it is that theory be well leavened with careful empirical analysis. That is the basic thrust of most of IFPRI’s research.

John W. Mellor
Washington, D.C.
June 1987
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SUMMARY

There is currently much debate over policies designed to achieve socially optimal population size or rates of growth and, more particularly, what such socially desirable goals should be. But little attention has been paid to the question of why members of the present generation care about future generations. Presumably they care because each of them, individually, cares about his or her progeny. If this is the case, it raises the possibility that individual choice, in contrast to a collectively imposed solution, may indeed achieve socially optimal results, at least according to some criteria of social optimality.

A minimal criterion of social optimality is that of so-called Pareto efficiency. Individual choice in a free-market context does, in general, lead to an equilibrium that is Pareto efficient. But there are circumstances that lead to a failure of laissez-faire to attain an efficient solution from the standpoint of members of the present generation. Such circumstances are illustrated in two cases. First, when bequests of parents to their children are a public good within the second-generation family, laissez-faire leads to market failure because parents may fail to take into account the additional utility that their bequest brings to the parents of their child’s spouse. A subsidy to bequests, coupled with a tax on children, can correct this failure. Second, if children have differing ability and parents cannot enforce transfers among siblings, market failure may occur because parents underinvest in their more able children in order to achieve greater equality of consumption among their offspring. Plato’s solution (in The Republic) was to abolish the family. A less drastic, second-best policy, however, would be a progressive income tax coupled with a subsidy to the bequest of financial assets. Public investment in education is shown to be redundant and a subsidy to education to reduce welfare.

To go beyond the efficient allocations from the standpoint of the present generation, it is necessary to introduce a social welfare function that aggregates, and therefore compares, the utilities of the present and future generations. In this report two are considered: the sum of individual utilities and the average of individual utilities. The analysis shows that the former always leads to a larger optimal population than the latter. When fertility is endogenous, a laissez-faire solution is well-defined, but it cannot be shown to lie between the results produced by the two social criteria, or, indeed, to bear any particular relation to them. When fertility is endogenous, a positive subsidy to second-generation consumption, and a child allowance (which may be negative) can be used to achieve a social optimum in a noncoercive, price-based manner.

This report explores the implications of the endogenous determination of fertility for certain issues in population policy. By endogenous fertility is meant something different from endogenous population size. It means that parents care about the numbers and welfare of their children and respond to economic constraints and opportunities when making choices affecting their children. The consequences of endogenous fertility for many issues of population policy are far-reaching. With respect to the socially optimal size of a population, does maximization of the sum of parents’ and children’s utilities lead to a higher rate of population growth than maximization of the per capita total? Also, does a laissez-faire solution (equivalent to maximizing a social welfare function that gives weight only to the utilities of the present generation) necessarily lead to a
higher rate of growth than maximization of per capita utility or to a lower rate than maximization of total utility? The reason for a possible ambiguity is found in the endogeneity of fertility for the current generation. Indeed, there is no laissez-faire solution unless children are directly or indirectly valued by parents. The report considers alternative noncoercive policies to support various allocations, such as child allowances and interest rate subsidies.

It also considers the implications of endogenous fertility for market failure, that is, the failure of laissez-faire allocation to achieve Pareto efficiency for the current generation. It is noted that two potential sources of externalities, diminishing returns and public goods, do not lead to market failure; since parents care about their children, the sources of externalities are internalized in parental fertility decisions. However, parental concern for the welfare of their children may give rise to other sorts of market failure, among which are those associated with the marriage of children and variations in the abilities of offspring.

When children are not valued for their own sake but only as a device for transferring resources from present to future consumption, that is, to provide security in old age, some researchers hypothesize that introduction of an alternative form of saving will reduce population growth. This theory is shown to be false when general equilibrium effects are taken into account. Furthermore, even if one ignores such effects, when fertility is endogenous because parents care about their children, relaxation of the constraint to saving in forms other than children creates a positive income effect, and parents may still bring more children into the world.

Endogenous fertility also has implications for policies influencing income distribution between generations, since such policies affect both the number and quality of individuals in successive generations. For example, even if poor people tend to substitute numbers of children for investments in child quality, positive child allowances may still be optimal for redistributing income within the current generation. The optimality of child allowances as a means of income redistribution may be affected, however, by the interaction of endogenous fertility with labor supply decisions. These issues are important for the incorporation of demographic elements into the optimal tax system.
INTRODUCTION

Malthus and the classical economists combined a very simple model of family decisionmaking with an equally simple model of the operation of the economy. In essays published in 1798 and 1830, Malthus foresaw for the family procreation without bound except possibly by "... a foresight of the difficulties attending the rearing of a family... and the actual distresses of some of the lower classes, by which they are disabled from giving the proper food and attention to their children."¹ For the economy, Malthus said that a high rate of capital accumulation induced by high profits—representing the difference between output and the rent of land (natural resources) and wages—permitted a continual increase in output and population, albeit at the cost of using land of increasingly poor quality. As a result of the model of family decisions, the standard of living of most people did not continue to rise but eventually fell. The Malthusian theory is a positive one: it purports to explain, under given circumstances, what will happen. However, the classical economists drew normative conclusions from the Malthusian theory, namely, that unbridled population growth was a bad thing because it resulted in ever lower per capita consumption until most people reached a subsistence level. Simplifying, one could argue that the classical economists implicitly assumed a social welfare function (see below) in which per capita utility matters but not the number in the population. This position can be referred to as Millian since it was John Stuart Mill who, more than anyone else, systematized and codified the classical tradition. In contrast, the utilitarians, represented by Bentham, held that the greatest good for the greatest number, that is, total utility, was the appropriate goal for society.²

Issues connected with population change have always been important in discussions of economic growth. Modern growth theorists in the tradition of Solow and Swan have developed theories of economic growth based on far more elaborate theories of the economy than the classical economists,³ but few theories of population growth and household decisionmaking have gone much beyond the Malthusian model.⁴ Although natural-resource constraints may be readily incorporated in theories of population growth through diminishing returns to scale in the variable factors,⁵ it is a constant proportional rate of exogenous population growth, perhaps aided and abetted by exogenous technological progress, that essentially drives the mechanism. While discussions of optimal rates of population growth or the size of a population often attempt to integrate an endogenously determined population in the model, none has examined the response to changes in the economy and changes in relative prices and costs of

⁴ For example, see J. D. Pitchford, Population in Economic Growth (Amsterdam: North Holland, 1974), pp. 1-10.
families in deciding how many children to have and what to invest in those children's health, welfare, and education. 6 This household response is endogenous fertility as distinguished from endogenous population.

The recognition, crucial to the understanding of long-term growth, that much investment in the economy is made in human beings rather than in physical capital and that fertility itself is shaped in important ways by economic considerations has led in recent years to a renewed interest in the economics of household decisions. It is in the household that decisions about consumption, savings, labor-force participation, migration, investment in human capital, and fertility are made. The theory of household decision-making in its modern form has been called the "new home economics." 7 It has developed principally from the work of Gary Becker, but most of its essentials are to be found in the earlier work of Margaret Reid, and it owes a good deal to Wesley Mitchell's insights in his essay on "The Backward Art of Spending Money." 8

Since Becker's analysis in 1960, the implications of endogenous fertility in the sense of parental altruism toward their own children, for consumption, labor supply, and household employment decisions have been explored extensively in the literature. 9 The purpose of this report is to examine the general equilibrium implications of endogenous fertility for a number of social issues related to population policy. These issues include the optimal size or rate of growth of population, real and false externalities, and issues of inter- and intragenerational income distribution. The concern is thus with the normative rather than the positive implications of endogenous fertility. Endogenous fertility simply means that parents care about the numbers and welfare of their children and respond to economic constraints and opportunities in making the choices affecting their children. It is remarkable that this simple and obvious concept has such far-reaching and significant normative implications. It is even more remarkable, however, that the idea that parents care about their children does not seem to have found a place in the current ethical and philosophical debates about optimal population.

In this analysis, the simplest possible formulation of endogenous fertility is adopted: in addition to their own consumption, the number of children and the utility of each child is assumed to enter the utility function of the parents. Thus, subject to whatever economic opportunities and constraints they face, parents are assumed to maximize their own utility functions (one per couple) in making choices about the number of children they have and how much and what to invest in them. Noncoercive tax and subsidy policies may be devised to affect these decisions; in the absence of such policies, a laissez-faire solution will generally exist. This implicitly assumes that the two parents in a family constitute one homogenous unit—the "household"—when making decisions on fertility, consumption, and investment. It is assumed that the household speaks with one voice so that it has a well-defined, consistent set of preferences that can be

9 Becker, "An Economic Analysis of Fertility."
used to explain its fertility, consumption, and investment (saving) decisions as the outcome of a process in which the household chooses the best among all the alternatives that it can afford. This does not necessarily imply that all costs and benefits from children are shared equally between the two parents. For instance, in a totally female-dominated society in which the male parent does not participate at all in household decisionmaking, there will still exist a well-defined set of preferences for the household, namely the set of preferences of the female parent. (Also see the discussion of interpersonal comparisons of utilities in Chapter 3.)
REVIEW OF WELFARE ECONOMICS

Most economic or social policies involve conflicts of interests among different members of society. Most frequently, some gain while others lose as a result of any particular social policy. Thus, welfare economics—the branch of economics dealing with normative issues—cannot escape the difficult task of weighing the gain of some members of society against the loss to others. Put differently, the design of a social policy typically involves interpersonal comparison of utilities (gains and losses).\(^\text{10}\)

One criterion that does not depend on such comparisons for different individuals is as follows: a measure that improves the well-being of some (or all) individuals without making anyone else worse off is socially desirable. Such an improvement is called a Pareto improvement. A related notion is Pareto efficiency. An allocation of economic resources (or, more generally, a social state) is said to be Pareto efficient if no Pareto improvement is possible. That is, an allocation is Pareto efficient if no reshuffling of resources can make some people better off without making at least one person worse off. Once the society attains a Pareto-efficient allocation, no proposed change can attract a unanimous vote. In general, the term "efficiency" as used by economists means Pareto efficiency.\(^\text{11}\)

The question naturally arises whether economic systems can attain Pareto efficiency. A system like that in the United States can be described as a market economy in the following sense: in the marketplace, self-motivated, utility-maximizing consumers express their demands for various goods and services. Similarly, self-motivated, profit-seeking firms express their willingness to supply these commodities. Competitive markets clear when prices are such that supplies meet demands. Adam Smith was the first to note that the outcome of this market-clearing process is efficient. He wrote picturesquely about the "invisible hand," which guides the many consumers and firms in a laissez-faire environment, each looking for its own benefit and with no coordination among them, to an outcome that is good for society. In modern language, market allocations, not always but under some conditions to be discussed later, are Pareto efficient.

An elementary explanation of this result is as follows. Suppose there is only one individual, whose demand for a particular commodity is represented in Figure 1 by the curve \(D = MB\). The demand curve (as well as the demand curves for all the other commodities valued by the individual) is the outcome of the maximization of utility.

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\(^{10}\) When one considers a household consisting of two parents to be the smallest unit to be reckoned with by society, one encounters a difficulty as to how to define "the utility" of the household. One possible solution is to define the household's utility as a function of the two utilities of the parents and to assume that the resources of the household are divided between its two members so as to maximize this function. This solution was offered in Paul A. Samuelson, "Social Indifference Curves," *Quarterly Journal of Economics* 70 (February 1956), in defining the concept of social utility.

\(^{11}\) It should be noted that the Pareto principle is unambiguously defined only for a fixed-size population. When the size of population itself is an endogenous variable, the principle is somewhat problematic: What is the meaning of hurting or making better an as yet nonexistent person? This issue is discussed extensively in Marc Nerlove, Assaf Razin, and Efraim Sadka, *Household and Economy: Welfare Economics of Endogenous Fertility* (New York: Academic Press, 1987), Chapter 6. This essay, except for Chapter 8, is restricted to the comparison of the welfare of the currently existing individuals, members of the present generation (who in turn care about the welfare of their offspring).
subject to the consumer’s budget constraint. This constraint is that expenditures on all commodities should not exceed income from all sources. The price along the demand curve indicates the amount of money the consumer is willing to pay for the marginal unit of the good. Therefore, the demand price is just the gross benefit/utility, measured in monetary equivalents, that the consumer extracts from the marginal unit. Hence the demand curve can be viewed as the marginal benefit curve, and denoted by MB. MB falls as the quantity increases. The sum of the marginal benefits derived from all the units up to a certain quantity, say, Q₁ (more precisely, the area under the demand curve), measures the total gross benefit. For example, the area OAHO₁ is the total benefit the consumer enjoys when consuming Q₁ units of the good.

Figure 1 also depicts the supply curve for this good. Elementary microeconomic theory says that the supply schedule is just the marginal cost curve, which is denoted as S = MC. Just as the area under the D = MB curve measures total benefit, the area under the S = MC curve measures total cost. For example, the total cost of producing Q₁ units is given by the area OFGQ₁.

The net benefit to society of producing and consuming the commodity in question is the excess of total benefit over total cost and is represented by the area between the demand and supply schedules. This is called net economic surplus. For example, the net economic surplus at Q₁ is FAHG. Adam Smith’s theorem says that net economic surplus is maximized at the competitive market-clearing price, P₀, where MB = MC, as can easily be seen from Figure 1.

Now, suppose there are many individuals and assume that the curve D = MB represents the aggregate demand (the horizontal sum of individual demands) for the good. Then the area under the demand curve represents the total benefits enjoyed by all members of society, and the area between the demand and supply curves represents the total net benefit accruing to all members of society. In this case it can still be concluded that the total net benefit is maximized at the competitive market-clearing price, P₀. This result, which essentially states that laissez-faire competitive market equilibria are efficient, is the first optimality theorem of welfare economics.

Notice also that if the government intervenes in the marketplace by, say, imposing an excise tax, the competitive equilibrium will no longer be efficient. To see this, observe that an excise tax raises the marginal cost curve of the firm by the amount of the tax rate, and a new equilibrium occurs at Q₂. Notice, however, that the cost of production to society does not rise as a result of the tax and hence the marginal cost is still represented, as before the tax, by the old MC curve. Thus the net economic surplus remains the area between the MC and the MB curves. Therefore, the net economic surplus at Q₂ is only FAKM, which is lower than the competitive surplus, FAB. Society loses KMB, called Harberger’s triangle. The excise tax thus creates a distortion. The cause of the distortion lies in the wedge that the tax drives between the true (social) marginal cost as given by the MC curve and the marginal cost from the firm’s standpoint, which also includes the excise tax. On the other hand, a tax that does not drive a wedge between private and social marginal costs or benefits does not distort. A lump-sum tax is an example of such a tax that preserves efficiency.

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12 For this statement to be exact, one must assume that the marginal utility of money income is constant so that the marginal benefits measured in monetary units are added together. These monetary units then represent the same amount of utility.

13 This result has been demonstrated within a rather limited partial equilibrium framework. Nevertheless the result holds, in a general equilibrium context, with many goods and factors of production.

The distribution of the total net benefits among the members of society depends on the underlying distribution of ownership of the society's economic resources (for example, labor, capital, land, and shares in firms' profits). This distribution of resources determines the distribution of individual demands. The areas under these demand curves represent individual gross benefits. Also, the demand curves together with the supply curves determine the market-clearing price. A redistribution of initial resource endowments will therefore change not only the market-clearing price, $P_0$, but also the distribution of net benefits. The "invisible hand" ensures that the new competitive equilibrium allocation is also Pareto efficient. Thus continuously redistributing initial endowments creates a continuum of market allocations that are all Pareto efficient. In fact, under certain conditions every possible Pareto-efficient allocation can be obtained by a proper redistribution of initial endowments and by letting the competitive markets clear.\(^{15}\) This result is the second optimality theorem of welfare economics.

To obtain additional insight into these distributional issues, suppose there are only two people, A and B, in society. Consider Figure 2. A distribution of net benefits (utilities) is a pair $(u^A, u^B)$, represented by some point in this figure. A point representing an efficient allocation is, by definition, a point from which no movements toward the northeastern boundary are feasible. Thus the locus of all possible Pareto-efficient points must be downward sloping, indicating that increasing $u^A$ inevitably involves decreasing

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\(^{15}\) Perhaps the most important among these sufficient conditions is nonincreasing returns to scale in production.
Figure 2—The Pareto frontier

\[ W(u^A, u^B) = W \]

\[ W(u^A, u^B) = W^* \]

\[ W(u^A, u^B) = \bar{W} \]

\( u^A \) and vice versa. The locus of Pareto-efficient points is called the Pareto frontier. Any point below this frontier is not efficient and any point above it is not feasible.

Figure 2 demonstrates the weakness, or the limited usefulness, of the Pareto-efficiency concept in comparing different allocations. For instance, C is an improvement over E in the sense of increasing a Pareto efficiency, and should therefore definitely be chosen over E. But the criterion of Pareto efficiency cannot be used to choose between C and H, because going from C to H improves the welfare of A, but decreases the welfare of B. The choice between C and H must therefore involve an interpersonal comparison of utilities (weighing the gain to A against the loss to B). In practice, most policy options involve exactly this kind of choice.

A social welfare function, \( W = W(u^A, u^B) \), which measures the welfare of society from each pair of utility levels \( (u^A, u^B) \), can be used to make choices involving interpersonal comparisons of utilities. A choice among allocations (pairs of utility levels) may be made according to the social welfare (measured by \( W \)) that society derives from each pair. Figure 2 describes society’s ranking over allocations (pairs of utility levels). This is done by drawing a map of social indifference curves. Welfare is constant along any such curve. Point K represents the highest welfare attainable by society. It is therefore the socially optimal allocation. Note that, by the second optimality theorem, this allocation is also the outcome of laissez-faire competition in the marketplace, provided a certain redistribution of the initial endowment of resources is made.
EXTERNALITIES AND PUBLIC GOODS

In the preceding chapter, the validity of the two theorems concerning the relationship between competitive equilibria and Pareto efficiency is qualified by certain conditions. These are principally the absence of externalities and the nonexistence of public goods.

Externalities

So far, it has been implicitly assumed that an action taken by any one of the agents (consumers or firms) in the market directly affects only that agent's utility or profit and that of no one else. In other words, all the costs and benefits resulting from the actions of that agent are fully perceived and accrue to that agent. We say these costs and benefits are fully internalized. In this case, self-interested individual decisions were shown to lead, under certain assumptions, to an efficient outcome, that is, a laissez-faire competitive equilibrium is Pareto-efficient.

However, there are many important instances where actions taken by one agent have effects that are external to that agent. The action of one agent may directly affect the utility or profit of some other agent (or agents). In such cases, it can be said that externalities exist. Perhaps the most famous example of externalities is that of the fable of the bees.16 The owner of an apple orchard produces, of course, apples. But as a by-product, apple trees also yield apple blossoms. A neighboring farmer raises bees to produce honey. However, his bees consume nectar from apple blossoms. In this case, the apple blossoms are an unpriced input into the production of honey. The action of the orchard owner generates an external effect on another agent, the honey producer, via the production of apple blossoms. The apple grower in this case does not capture the full benefit of his activity because he sells only the apples but not the apple blossoms. His actions, motivated by maximizing his own profit, overlook the benefit to the honey producer. (And, of course, the bees pollinate the apple trees, which is an unpriced benefit to the apple grower.) In any case, the marginal private benefit from apple production (accruing to the orchard owner) falls short of the marginal social benefit, which is the sum of the marginal private benefit of the orchard owner and the marginal benefit of the honey producer. Thus the action of the apple producer, while "correct" from his perspective, is "wrong" from society's standpoint. A market failure occurs: a competitive laissez-faire market fails to achieve an efficient output of apples. In the apple blossom/honey bee example, the external effect is beneficial and is, therefore, called a positive externality or an external economy. Of course, in another case the external effect might be harmful, in which case it is called a negative externality or an external diseconomy. An example is a chemical firm that dumps its waste into a river, which reduces the catch of a fisherman downstream.

Figure 3 shows graphically why a market failure arises in the apple-blossom/honey bee example. The curve MPB_A represents the marginal private benefit (revenue) accru-

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Figure 3—The apple-blossom/honeybee example

The apple grower. His marginal cost curve is the curve MC. The competitive decision about how many apples to produce is made solely by the apple grower. His private profit is maximized at $Q^0$, where his marginal private benefit is equal to his marginal cost (his profit is given by the area FEC). $Q^0$ is thus the competitive output of apples. However, apples also benefit honey production. The marginal value product of apples (via apple blossoms) in honey production is depicted by the curve, $MPB_H$. Thus the marginal social benefit of apples is given by the curve MSB, which is the vertical sum of $MPB_A$ and $MPB_H$. The efficient output of apples is, therefore, $Q^*$, where $MSB = MC$ and the total net benefit to society is $FMN$. (Compare this to $FMKC$, which is the net social benefit at the competitive equilibrium.) The cause of the market failure can be easily pinpointed: the apple producer does not take into account (and justifiably so from his standpoint) the benefit, $MPB_H$, accruing to the honey producer.

This discussion immediately suggests two kinds of remedies for the market failure. The apple grower can be induced to produce more if his MPB curve is raised by a subsidy. If the per-unit subsidy is equal to $NR = TQ^*$ (which is exactly the marginal value product of apples in honey production at the efficient level of output, $Q^*$), then the apple grower will produce $Q^*$ because his marginal private benefit curve will now be the curve labeled “$MPB + Pigouian Subsidy,” and it intercepts his MC curve at $Q^*$. Such a subsidy is called a Pigouvian subsidy. Another remedy is for the two producers to merge. The marginal private benefit of the new firm will be just MSB and it will produce $Q^*$. In this case the external effect is fully internalized and no market failure arises.

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17 The subsidy must be funded by nondistortionary taxes. In the case of an external diseconomy, the remedy will be a Pigouvian tax; the revenue from such a tax should be used in a nondistortionary fashion.
Public Goods

Most goods are ordinary private goods in the sense that they can be parcelled out among different individuals or, at least, among different families. But there are many goods, such as national defense, television, or radio broadcasts, that "...all enjoy in common the sense that each individual's consumption of such a good leads to no subtraction from any other individual's consumption of the good."18 Such goods are called public goods.

From the point of view of the individual consumer, consumption of a public good entails utility in exactly the same way as consumption of an ordinary private good does. Therefore, as before, one can derive a demand curve for a public good for each individual. This demand curve is also the marginal benefit curve. Consider two individuals, A and B, whose demand curves for a public good are, respectively, the curves $D_A = MPB_A$ and $D_B = MPB_B$ in Figure 4. In this case, any output of the public good is consumed simultaneously by both A and B. Hence the marginal social benefit (MSB) at each level of output is the (vertical) sum of the marginal private benefit accruing to A ($MPB_A$) and the marginal private benefit accruing to B ($MPB_B$).19 Plotting the marginal cost curve as the curve MC in Figure 4, the efficient level of output of the public good can be seen to be $Q^*$, where the $MSB = MPB_A + MPB_B = MC$. This condition of Pareto-efficiency is known as the Lindahl-Samuelson condition.

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19 Issues of income distribution are ignored because the analysis concerns only Pareto efficiency.
Can this output be reached in a competitive market? Generally, the answer is no. To induce a firm to produce \( Q^* \), it must be given the price \( P^* = MSB^* \). Individuals A and B will both purchase the quantity \( Q^* \) if they pay the prices \( P_A^* = MPB_A^* \) and \( P_B^* = MPB_B^* \), respectively. That is, each individual must pay a price equal to that person's marginal private benefit (at the efficient level of output) and the firm must pocket the sum of all the individual prices. Such an equilibrium is called a Lindahl equilibrium. However, the problem of "free riding" makes the Lindahl equilibrium virtually impossible to achieve. Usually once the public good is produced, exclusion of an individual from consuming it is prohibitively costly. Realizing this, each individual attempts to free-ride and the firm will not be able to collect much revenue for its product. Therefore, Lindahl equilibria are not viable. Of course, the government can provide public goods paid for by imposition of taxes. Whether it can provide efficient levels of public goods depends on its ability to measure the true individual valuations (the MPB curves) of the public goods.
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ENDOGENOUS FERTILITY AND POTENTIAL MARKET FAILURE: FALSE ISSUES

So far, it has been assumed that utility is derived only from the consumption of ordinary commodities. To come more to the point of this essay, it is now assumed that utility can be generated from consumption of another "good": children. It is assumed that parents are altruistic toward their own children so that they extract utility from the number and from the welfare of their offspring. Furthermore, the decision-making problem of the consumer is now extended to include the number and welfare of offspring (endogenous fertility). This chapter examines two potential sources of market failure that may arise when fertility is endogenous.

First, if there are pure public goods, such as national defense, basic research, and weather forecasts, the per capita costs of providing these goods fall as the population size is increased. Since all enjoy these goods at no additional cost, the market may fail in relation to population size, resulting in the inefficiency of laissez-faire. Second, a fixed resource, such as land, which must be combined with labor to produce goods for consumption, could lead to Malthusian diminishing returns to a larger population size. This suggests a potential source of external diseconomies and market failure in relation to population size. It is remarkable that neither of these two potential sources leads to market failure when fertility is endogenous; competition leads to Pareto efficiency from the standpoint of the present generation.\(^{20}\)

Consider, for the sake of simplicity, a two-period model with one parent in the first period. It is assumed that the supply of land is fixed, and that the supply of labor per capita is fixed as well (that is, there are no labor-leisure decisions). Land is used in each period, together with labor, to produce a single good that can be used either as private consumption \((c^1)\) or as public consumption \((p^1)\) in period \(i = 1, 2\). \(c^1\) is consumption by the parent in the first period, while \(c^2\) is consumption by each child in the second period. Due to the Malthusian assumption that the amount of land is fixed, there is a diminishing marginal product of labor. Assuming that the labor endowment is one unit, output is \(f(l)\) in the first period, where \(f\) is a production function that, it can be assumed, exhibits a diminishing marginal product, \(f' > 0\) and \(f'' < 0\). The parent in the first period bears \(n\) children. Therefore, output is \(f(n)\) in the second period.

The consumption possibilities of this economy can be described by the following two resource constraints:

\[
\begin{align*}
    c^1 + p^1 + b &= f(1) \quad (1) \\
    nc^2 + p^2 &= b + f(n) \quad (2)
\end{align*}
\]

where \(b\) is the quantity of consumption transferred from the parent in the first period to children in the second period. Equation (1) states that total output in period 1, \(f(1)\),

\(^{20}\) In fact, the Malthusian external diseconomy of larger population size should not cause any market failure even when fertility is not endogenous. The reason is that this externality is a pecuniary externality rather than a technological externality. Only the latter type of externality causes a market failure, as explained in Chapter 4.
is used for private consumption, $c^1$, public consumption, $p^1$, and bequests, $b$. Equation (2) has a similar interpretation. Implicitly it is assumed that the private good can be stored from the first to the second period without cost or returns. These two constraints are combined to yield a single constraint:

$$c^1 + nc^2 + p^1 + p^2 = f(1) + f(n),$$

which specifies the overall resource constraint faced by society.

The government provides the public goods in each period and finances them by a lump-sum tax ($T$), which is imposed on the parent and all the parent’s progeny; that is, the dynasty as a whole. Notice that in this model a head tax is not a lump-sum tax because the number of children is endogenous; hence, a head tax should be regarded as an excise tax on children. This is the reason for imposing a fixed tax, $T$, on the whole dynasty rather than a head tax on each of its members. The government budget constraint is written as:

$$p^1 + p^2 = T.$$  

The government is thus restricted to a balanced budget over the whole horizon rather than at each period.21 Any arbitrary pair ($p^1$, $p^2$) of public good provisions can be considered, including the efficient pair. It will be shown that there is no market failure despite a seemingly noninternalized benefit from the lower cost per capita of providing the public good that a greater population size has. Since any pair of public good provisions is considered, the result holds, therefore, whether or not the government provides the efficient pair.

The parent in period 1 maximizes utility, which depends on private consumption ($c^1$ and $c^2$), public consumption ($p^1$ and $p^2$), and the number of children ($n$):

$$u = u(c^1, c^2, p^1, p^2, n).$$

Obviously, the values of $p^1$ and $p^2$ are not chosen by the parent but are provided by the government. Thus, the parent maximizes his or her utility with respect to $c^1$, $c^2$, and $n$, subject to the budget constraint

$$c^1 + nc^2 = w^1 + nw^2 + \pi^1 + \pi^2 - T,$$

where $w^i$ is the equilibrium wage rate in period $i$, and $\pi^i$ is the equilibrium land rent ($i = 1, 2$) that accrues to the owners of the land, that is, to the parent in period 1 and to the children in period 2. The parent who cares about his or her children makes plans for their consumption, taking into account their earnings ($nw^2$) and the land rent ($\pi^2$) accruing to them in the second period. The parent also takes into account the entire tax bill ($T$) of the dynasty. The receipt by the children as a group of both labor income and land rent is really the key to the conclusion here.

To examine whether the competitive equilibrium is efficient, consider Figure 5. In period 1, society consists of the single parent then living. To focus attention on the

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21 In fact, it does not matter here whether the government is restricted to a balanced budget at each period or only over the whole horizon because the parent cares for his or her children. As long as the bequest is strictly positive, the parent can use the bequest to undo any intergenerational distribution of taxes by the government.
issue at hand, it is assumed that the objective of society coincides with the objective of this parent, which is to maximize his or her utility (alternative social objectives will be discussed below). Hence the marginal social benefit (MSB) from the number of children coincides with the private one (MPB). Next, consider the marginal social cost of children. As can be seen from the overall resource constraint of society, equation (3), the marginal cost of $n$ is $c^2 - f'(n)$, where $f'$ is the marginal product of labor. The marginal child consumes $c^2$ but also produces a marginal product of $f'(n)$. Hence the parent's net marginal cost is $c^2 - f'(n)$. Since the marginal product is diminishing, the curve $MSC = c^2 - f'(n)$ is upward sloping.

Now consider the marginal private cost of $n$. It can be seen from the parent's budget constraint, equation (6), that the marginal child consumes $c^2$, but also earns a wage of $w_2$. Hence the marginal private cost of children to the parent is $MPC = c^2 - w^2$. Since the parent and the children are assumed to behave as perfect competitors, that is, as price-takers and wage-takers, they consider $w^2$ to be constant. Hence, the MPC curve is horizontal.

Thus the equilibrium number of children is $n^*$. The key point to observe here is that profit-maximization requires that labor (children) be employed up to the point where the marginal product is equal to the wage. Since $w^2$ is the equilibrium wage, therefore, $f(n^*) = w^2$. Hence, MSC equals MPC at the equilibrium number of children, $n^*$. Therefore, MSC is equal to MSB at $n^*$, so that $n^*$ is also the efficient number of children.
Thus public goods and Malthusian fixed land do not cause market failure from the standpoint of the present generation when fertility is endogenous. On the other hand, endogenous fertility may create a new difficulty: the competitive equilibrium may fail to exist. The ability to control fertility may lead a mother who initially has a negligibly small effect on prices to believe that she can increase her utility ad infinitum by increasing her fertility rate without bound. But if she adopts such a course of action, she will no longer have negligibly small effects and her behavior will start to influence prices, thus breaking down perfect competition. This point can be clearly seen if the parent's budget constraint, equation (6), is rewritten as

\[ c^1 = w^1 + n(w^2 - c^2) + \pi^1 + \pi^2 - T, \]  

(7)

Recall that the parent is unable to affect prices (w^2 is a constant parameter). Hence she must believe that by setting (if possible) c^2 below w^2 (thereby making the net return from children, namely w^2 - c^2, positive), she can increase n, c^1, and, consequently, u to infinity. But for the economy as a whole, w^2 is not constant; it is equal to the marginal product of labor, f'(n), which is diminishing. Therefore, no constant w^2 can be consistent with an arbitrarily large n. A competitive equilibrium may fail to exist.

Nevertheless, the possibility of the nonexistence of an equilibrium seems not to be plausible in practice. For example, in this case, an equilibrium can be restored by assuming that there is some cost of raising children (where the marginal cost is positive and increasing). In a reduced form model, this cost could be built into the utility function, so that beyond a certain level n stops being a "good" and starts being a "bad," implying that there will be then a disutility from children. Alternatively one may assume that c^2 cannot be lowered below a certain subsistence level. Thus if w^2 is already at or below this subsistence level, then the parent can no longer reduce c^2 below w^2 and then increase c^1, n, and u to infinity. Notice that in this case a competitive equilibrium requires the wage rate (w^2) to fall to or below the subsistence level. In this case, the Malthusian hypothesis is partially confirmed even with endogenous fertility: the wage falls to the subsistence level, but there is no presumption that consumption (c^2) must follow suit.
ENDOGENOUS FERTILITY AND POTENTIAL MARKET FAILURE: REAL ISSUES

Although the obvious cases of externalities when fertility is endogenous do not appear to occur, two real sources of market failure arise from bequests in a model in which parents care about their children. In the absence of such concern, parents will never transfer (bequeath) anything to their children in a world of perfect foresight and certainty about the time of death.

Marriage and Bequests

Consideration of bequests and of marriage suggests a potential source of market failure as follows: if bequests benefit both partners in a marriage (as a public good within a marriage), parents may fail to include benefits to other children's parents in deciding on the amount of bequests to make to each of their own children. This suggests that there will be external economies generated by bequests within a marriage.

For the sake of simplicity, assume that there are only two families in the current generation and only two generations (periods), so that

\[ c_i \quad \text{the consumption of the } i\text{th family in the first period}, \]
\[ n_i \quad \text{the number of children of the } i\text{th family}, \]
\[ b_i \quad \text{the bequest per child of the } i\text{th family}, \]
\[ k_i \quad \text{the resources available to the } i\text{th family for consumption and bequest, and} \]
\[ i = 1, 2. \]

The total bequest of two children who marry one another will be the sum of the bequests to each child, that is, \( b_1 + b_2 \). This sum is also assumed to be the consumption of the second generation. For the sake of simplicity, assume that the two families bring the same number of children into the world, so that the number of children available to marry each other will be identical.

Consider first the parents of family 1 in the first period. These parents derive utility from their own consumption \( (c_1) \), the number of their children \( (n_1) \), and the consumption, \( b_1 + b_2 \), of the newly formed family of each child in the second period:

\[ u^1 = u^1(c_1, n_1, b_1 + b_2). \quad (8) \]

Observe that \( b_2 \) is bequeathed by the parents of family 2, and is beyond the control of the parents of family 1. Therefore, the latter treats \( b_2 \) as a constant. They choose only \( c_1 \), \( n_1 \), and \( b_1 \) so as to maximize equation (8), subject to the budget constraint:
\[ c_1 + b_1 n_1 = k_1. \]  

Similarly, the parents of family 2 choose \( c_2, n_2, \) and \( b_2 \) (treating \( b_i \) as an exogenous parameter), so as to maximize their utility:

\[ u^2 = u^2(c_2, n_2, b_1 + b_2), \]

subject to their budget constraint,

\[ c_2 + b_2 n_2 = k_2. \]

The competitive equilibrium obtained by the above maximizations by the two families can be denoted by

\[ c_1, c_2, \bar{n}_1 = n_2 \equiv \bar{n}, \bar{b}_1, \bar{b}_2. \]

Now to examine whether this allocation is Pareto efficient. It will be shown that there are external economies associated with the amounts of bequests and hence the equilibrium allocation cannot be efficient. Consider Figure 6. The per child bequests made by the parents of family 1 (\( b_i \)) are plotted on the horizontal axis and the units of the all-purpose good are plotted on the vertical axis. From the budget constraint of parents 1, equation (9), it can be seen that the marginal cost of \( b_1 \) is \( MPC = \bar{n}_1 \), because increasing the bequest made to each child by one unit increases the total cost by the equilibrium number of children, \( \bar{n}_1 \). Increasing \( b_1 \) by one unit increases utility by the marginal utility of the children’s consumption, \( u_1^1 \). To express this marginal utility in units of the good, divide it by the marginal utility of the parents' consumption, \( u_1^1 \). Hence, the marginal private benefit to the parents of family 1 is \( MPB^1 = u_1^1/u_1^1 \). Therefore, the unconstrained equilibrium amount of \( b_1 \) is \( \bar{b}_1 \), where \( MPC = MPB^1 \).

Now to look at the marginal social cost and benefit curves in the same diagram. The cost of a bequest to society is the same as to the parent, that is, \( MSC = MPC = \bar{n}_1 \). However, \( b_1 \) also benefits the parents of family 2 because it increases consumption by their children who marry the children of the parents of family 1. Hence, the marginal private benefit of \( b_1 \) to the parents of family 2 is \( MPB^2 = u_2^2/u_2^2 \). The marginal social benefit of \( b_1 \) is the sum of \( MPB^1 \) and \( MPB^2 \): \( MSB = MPB^1 + MPB^2 = u_1^1/u_1^1 + u_2^2/u_2^2 \). Hence, the efficient level of \( b_1 \) will be \( \bar{b}_1 \) and not \( \bar{b}_1 \). A market failure exists.

The above discussion also suggests the Pigouvian remedy: MPC should be lowered by a proper subsidy to bequests so that it will intersect \( MPB^1 \) at the efficient level of bequest, \( b_1^* \). The rate of the subsidy (\( s^* \)) should be exactly equal to the proportion of the external effect in the MSB, \( u_2^2/u_1^1(u_1^1/u_1^1 + u_2^2/u_2^2) \), at the efficient level of bequest, \( b_1^* \). A similar subsidy should be applied to \( b_2 \) to induce the parents of family 2 to bequeath an efficient amount of \( b_2^* \).

In the standard case of externalities discussed in Chapter 4, the Pigouvian subsidy was all that was needed. Here things are a bit different. The budget constraints given by equation (9) or (11) show that \( b \) is the price of \( n \) because each additional child costs his or her parent the amount \( b \) bequeathed to him or her. (Similarly, \( n \) is also

\[ \text{Note:} \]
Figure 6—Marginal private and social benefits and costs of a bequest

\[ \text{Units of the Good} \]

\[ f_1 \]

\[ \text{MPC} = f_1 = \text{MSC} \]

\[ \text{MSB} = \text{MPB}^1 + \text{MPB}^2 = \frac{u_1^2}{u_1^1} + \frac{u_2^2}{u_2^1} \]

\[ \text{MPB}^1 = \frac{u_1^3}{u_1^1} \]

\[ \text{MPB}^2 = \frac{u_2^3}{u_2^1} \]

\[ b_1 \]

\[ b_1^* \]

\[ \text{Family 1's Bequest (} b_1 \text{)} \]

The price of \( b_1 \). Hence, subsidizing \( b \) distorts the parents' decision about the number of children to have because it lowers their private cost (but not the social cost). For instance, the budget constraint facing the parents of family \( b \) now becomes

\[ c_1 + b_1(1 - s) n_1 = k_1. \]  \hspace{1cm} (13)

To correct this distortion, children must be taxed directly so that the marginal private cost of a child will equal the marginal social cost. To find the appropriate amount of the tax, consider Figure 7, where the optimization of the number of children for the parents of family \( 1 \) is considered. The marginal private benefit of \( n_1 \) is the marginal utility of \( n_1 \), that is, \( u_1^1 \). Expressed in terms of units of the all-purpose good, it is \( \text{MPB} = \frac{u_1^2}{u_1^1} \). This is also the marginal social benefit of \( n_1 \), \( \text{MSB} \), since the number of children in family \( 1 \) does not affect the welfare of the other family. The marginal cost of \( n_1 \), to the parents of family \( 1 \) is the cost of the bequest to their children, which is \( \text{MPC} = b_1^*(1 - s^*) \). However, the marginal cost of \( n_1 \) to society is just \( b_1^* \), that is, \( \text{MSC} = b_1^* \). Therefore, to induce the parents of family \( 1 \) to bear the efficient number of children, \( n_1^* \), a tax of \( b_1^* s^* \) must be put on each child.
Figure 7—The marginal private benefits versus the marginal private and social costs of a child

In many parts of the world parents do, in fact, bargain with each other about what they will give their children. The economics of dowry and bride price, which has been considered in discussions of polygamy, is related to this discussion.23

Transfers Among Siblings

When children have different abilities, investments in their human capital are not equally productive. If parents cannot enforce transfers among their children, an egalitarian attitude may lead to inefficient investment in human and nonhuman capital. For example, the parents may be led to invest too much in the human capital of low-ability children so that they will be equal (in their utility) to their more able siblings. Thus a second source of potential market failure arises from the inability of parents to control the actions of their offspring after a certain point. In particular, they cannot enforce transfers among siblings. Parents who care about their children may wish to transfer resources to them. These transfers can take several forms, such as direct transfers of

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consumption (bequests), or indirect transfers made by investing in the human capital of the children, investments that increase the future consumption-possibility sets of the children. The most efficient method of transfer may depend on the specific characteristics of the child. Thus, parents may wish to use different methods of transfer for different children. Furthermore, it may be more efficient to make transfers (via investment in human capital) to only some of the children and force them later in life to make transfers to the other siblings. But this mode of transfer to children depends on the parents' ability to enforce the required transfers. This poses a difficulty that cannot be eliminated by appeal to Becker's "rotten-kid" theorem or by appeal to vaguely defined social norms.24 Becker and Tomes note the difficulty but suggest in passing that "...social and family 'pressures' can induce...children to conform to the terms of implicit contracts with their parents."25 Such norms might be effective in some circumstances in some societies but they have certainly not generally been effective even in ancient societies (as the biblical episode of Cain and Abel attests), let alone in modern societies.

The most important case in which equal transfers to siblings are not efficient even for parents conscious of equity among their children is when children differ in their abilities. It might be most efficient to invest only in the human capital of the children with greater abilities if parents could guarantee that these children would later on transfer part of the return to this investment to their less able siblings. However, if the parents cannot enforce transfers among siblings, then they may not be able to take advantage of high rates of return to investment in the human capital of their more able children. In this case, transfers in the form of investment in human capital from parents to children will be too low relative to bequests in the form of physical capital. Moreover, the investment in human capital will be inefficiently allocated among the children in the sense that the rates of return are not the same for all children.

A social planner who can identify ability can devise a system of taxes and transfers based on ability in order to achieve an efficient allocation of resources. However, when identification of more able and less able children is impossible or prohibitively costly to all except for parents themselves, a Pareto-efficient solution to the problem of optimal investment in human capital and bequests cannot be achieved.

It can be shown that a linear tax on earned income and a subsidy to inheritance are Pareto improvements even though they do not lead to Pareto efficiency. They are, therefore, second-best corrective policies. Such policies make the parents better off because they redistribute income from more able to less able siblings and allow parents to allocate the investments in human and physical capital that they make on their children's behalf more efficiently. Other policies, such as public investment in human capital or a tax/subsidy for education, can be shown to be Pareto inferior relative to the laissez-faire solution. Public investment in human capital (for example, free education) is redundant as long as parents invest positive amounts in children of all abilities.

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24 Gary S. Becker, "Altruism, Egoism and Genetic Fitness: Economics and Sociobiology," *Journal of Economic Literature* 14 (September 1976): 817-826; and Jack Hirschleifer, "Shakespeare vs. Becker on Altruism: The Importance of Having the Last Word," *Journal of Economic Literature* 15 (June 1977): 500-502. The "rotten-kid" theorem states essentially that when parents are altruistic in the sense of transferring positive resources to their children, the latter have the incentive to act as their parents wish—altruistically—toward each other, even if they are really purely egotistical. The theorem clearly does not apply in this case since the resources would have already been transferred at the time of the desired action.

because parents can always undo the effects of such a policy by reducing their investments in the human capital of their children dollar for dollar. Instead of direct government investment in human capital, a subsidy to education might be considered. Such a subsidy in the first period must be financed by a tax in the same period if the government cannot transfer resources from the future to the present. Moreover, it creates a distortion by artificially lowering the cost of education to the parents. This distortion can be shown to lower the welfare of the parents.

Interestingly, an income tax is proposed here as a partial remedy to the inefficiency of the laissez-faire solution that arises when parents cannot enforce transfers among their children. Such a tax can be shown to improve the efficiency of the laissez-faire outcome. Recall that a progressive income tax is usually supported on equity grounds. It is viewed as a second-best tool of redistributing income from the rich to the poor, as in the optimal income-tax literature.26 There, a smaller pie is traded for a better (more equal) division of it by means of the income tax. In the present case, however, the income tax is justified on pure efficiency grounds!

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7

CHILDREN AS A CAPITAL GOOD

This chapter considers the "old-age security hypothesis," which essentially views children as a capital good. In the words of T. W. Schultz, "children are 'the poor man's capital'" in developing countries. Becker writes that "...it is possible that in the mid-nineteenth century children were a net producer's good, providing rather than using income." Neher and Willis develop the idea that parents in less-developed countries are motivated, in part, to bear and rear children because they expect children to care for them in old age.

The old-age security hypothesis states that better access to capital markets unambiguously reduces the demand for children because children are then a less important means of transferring income from the present to the future. For instance, Neher writes that "...the good asset (bonds) drives out the bad asset (children)."

A Simple Model of Old-Age Security With No Capital Markets

Suppose that parents live for two periods. There is a single all-purpose good that is produced by labor alone. Each parent is endowed with an amount of labor capable of producing \( K_1 \) units of this all-purpose good. A parent has children during the first period, each endowed with an amount of labor capable of producing \( K_2 \) units of the good in the second period, when the child grows up. Each child consumes \( x_1 \) units in the first period and \( x_2 \) units in the second period. For the moment, assume that the parents do not care about the welfare of their children. They merely view them as a capital good intended to provide them with old-age consumption in the second period of their lives when they can no longer work. Consequently, the utility of either parent is assumed to depend only on \( c \) and \( \bar{c} \), his or her first-period and second-period consumption, respectively:

\[
u = u(c, \bar{c}).\]

Thus, \( x_1 \) and \( x_2 \) are assumed to be exogenously given (at conventional or subsistence levels).

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29 Becker, "An Economic Analysis of Fertility."
31 Neher, "Peasants, Procreation, and Pensions."
A parent can then use the output he or she produces in the first period for consumption (c), for investment in n children (nx), and for investment in physical or financial capital (S). Thus a parent faces the following budget constraint in the first period:

$$K_1 = c + nx + S.$$  \hspace{1cm} (15)

The investment in each child yields a return of $K_2 - x_2$ units of the all-purpose good in the second period. This return is simply the output of the child less its consumption. It is assumed that $K_2 - x_2 > 0$, for otherwise it does not pay to invest in children. Thus the parent's consumption in the second period (c), is given by

$$\bar{c} = n(K_2 - x_2) + (1 + r)S,$$  \hspace{1cm} (16)

where r is the real rate of interest.

In this subsection, it is assumed that no capital markets exist so that children become the sole form of capital and are the only means of transferring consumption from the present to the future. Consequently, set $S = 0$.

Solving equation (15) for n,

$$n = (K_1 - c)/x_1$$  \hspace{1cm} (17)

and equation (16) for n,

$$n = [\bar{c} - (1 + r)S]/(K_2 - x_2),$$  \hspace{1cm} (18)

equating equation (17) to equation (18) (recalling that $S = 0$) yields the parent's consumption-possibility frontier (between $\bar{c}$ and $\bar{c}$):

$$\bar{c} = (K_2 - x_2)(K_1 - \bar{c})/x_1.$$  \hspace{1cm} (19)

The interpretation of equation (19) is straightforward: second-period consumption for the parent is maximized by consuming nothing in the first period ($c = 0$) and investing all of the endowment, $K_1$, in $K_1/x_1$ children, which, in turn, yields $(K_2 - x_2)(K_1/x_1)$ in the second period. Notice also that $(K_2 - x_2)(K_1/x_1)$ is the future value of the parent's lifetime income measured in units of future consumption. A unit of present consumption ($c$) is a unit of forgone investment in children. Therefore it has an opportunity cost in terms of future consumption. This cost, $(K_2 - x_2)/x_1$, is the return to a unit of investment in the all-purpose good in children: investing one unit of the all-purpose good in children means bearing $1/x_1$ children who each yield $K_2 - x_2$. The parent's consumption-possibility frontier is depicted in Figure 8.

The parent chooses that point on the consumption-possibility frontier that maximizes his or her utility function. This is the point $(\bar{c}^*, \bar{c}^*)$ in Figure 8 where an indifference curve, $u(\bar{c}, \bar{c}) = \text{constant}$, is tangent to the consumption-possibility frontier. Once the parent chooses the optimal consumption bundle $(\bar{c}^*, \bar{c}^*)$, the optimal number of children $(n^*)$ is determined from equations (17) or (18):

$$n^* = (K_1 - \bar{c}^*)/x_1 = \bar{c}^*/(K_2 - x_2).$$  \hspace{1cm} (20)

The following example illustrates the result. Suppose the utility function (equation [14]) is of the Cobb-Douglas form
Figure 8—Equilibrium demand for consumption over two periods

\[
\frac{K_1(K_2 - x_2)}{x_1} = constant
\]

\[\tilde{c} = \frac{K_1(K_2 - x_2)}{x_1} - \frac{(K_2 - x_2)}{x_1} \tilde{c}
\]

\[u(\tilde{c}, \tilde{c}) = \tilde{c}^\alpha (1 - \tilde{c})^{1 - \alpha}, \quad (21)
\]

where a fraction, \(\alpha\), of lifetime income, \(K_1(K_2 - x_2)/x_1\), is spent on present consumption (\(\tilde{c}\)) and the remaining fraction \((1 - \alpha)\) is spent on future consumption (\(\tilde{c}\)). Consequently, the parent chooses

\[c^* = \alpha\{K_1(K_2 - x_2)/x_1\}/\{(K_2 - x_2)/x_1\} = \alpha K_1; \quad (22)
\]

\[\tilde{c}^* = (1 - \alpha)\{K_1(K_2 - x_2)/x_1\}; \quad \text{and} \quad (23)
\]

\[n^* = (K_1 - \tilde{c}^*)/x_1 = [(1 - \alpha)K_1]/x_1. \quad (24)
\]

**A Capital Market With an Exogenously Given Interest Rate and No Borrowing**

Now drop the constraint that \(S = 0\). Suppose there is a capital market in which parents can invest their savings. There is now an alternative to children for transferring present to future consumption. But no borrowing is yet allowed, that is, \(S\) can now be positive, though it still cannot be negative: \(S \geq 0\). The real return to savings is \(r\).\(^{32}\)

\(^{32}\) One can envision this as opening access to an international capital market with a given rate of interest in which the residents of the small underdeveloped country can invest their savings but cannot borrow.
Recall that for the moment the parent does not derive utility from the number or welfare of his or her children. Therefore, parents will not invest in children if the return that they yield, \((K_2 - x_2)/x_1\), is lower than the alternative return, \(1 + r\), in the capital market. Assume that \(K_2\), \(x_2\), and \(x_1\) vary by family. Parents will choose to have no children if \(x_2\) or \(x_1\) is sufficiently high or \(K_2\) is sufficiently low, so that

\[
(K_2 - x_2)/x_1 < 1 + r.
\]

(25)

Instead, they will transfer consumption from the present to the future via the capital market (that is, they will choose a positive \(S\)). On the other hand, parents for whom the inequality is reversed, so that

\[
(K_2 - x_2)/x_1 > 1 + r,
\]

would actually like to borrow in the capital market at the low rate of interest, \(r\), in order to invest in the high-yield asset, children, but they cannot. Hence, they will choose to make \(S = 0\). These parents are not affected by the introduction of this one-sided (lending only) capital market. Consequently, they will choose the same number of children as before.

Since if there is a capital market some families (those for whom equation [25] holds) will choose to have no children, while some families (those for whom equation [26] holds) will choose to have the same number of children as they would if there were no capital market, it follows that the total population must be smaller with a capital market than without one. This is the essence of the old-age security hypothesis.

The Old-Age Security Hypothesis Reconsidered: An Endogenously Determined Interest Rate

The analysis of the preceding section demonstrates that the old-age security hypothesis is a partial-equilibrium result: the interest rate, being exogenously given, did not clear the capital market; some families had positive savings while others who wanted to dissave were constrained not to do so. In this section, a perfect capital market is introduced in which people can both lend and borrow and in which the interest rate is determined to clear the market at equilibrium. Total savings equal total dissavings: the total supply of funds of those who save is equal to the total demand for funds of those who dissave.

In this case some families may indeed, as in the preceding section, choose to have no children if the capital market offers them an investment opportunity with a higher yield. But other families may now use the capital market for borrowing in order to invest more in children. Thus the introduction of a perfect (two-sided) capital market may well increase rather than decrease the number of children, contrary to the old-age security hypothesis. In the absence of a capital market, all families transfer resources from the present to the future through their children. With a capital market, only families with high rates of return on children (relative to the interest rate) continue to use children as a means of transferring resources from the present to the future. The other families with low rates of return on their children can also enjoy these high rates of return on children by lending to the families that have them and letting them invest in children. Thus families with a high rate of return on children invest in children not only for themselves, but also for families with a low rate of return. The introduction
of a capital market allows the economy to use children as a capital good more efficiently. Consequently, the economy may invest more in children. This is exactly what happens in the next example.

Suppose that there are only two types of families (A and B) who both have the Cobb-Douglas utility function of equation (21). The two families have the same endowments (K, and K₂), the same second-period child consumption (x₂), but different first-period child consumption (x₁ and x₁'). Assume that x₁' > x₁, so that the return on investment in children is higher for family B than for family A:

\[
\frac{(K_2 - x_2)}{x_1'} > \frac{(K_2 - x_2)}{x_1'}. 
\]

The earlier example showed that in the absence of a capital market the number of children in each family (see equation [24]) is given by:

\[
n^* = \left(\frac{(1 - \alpha)K_i}{x_1}\right), \tag{27}
\]

where i = A, B. (Note that the family with the higher return on children chooses to have more children.) The aggregate number of children in this case (N'), can be found from equation (27):

\[
N = n^A + n^B = (1 - \alpha)K_i\left(\frac{1}{x_1'} + \frac{1}{x_1'}\right) \tag{28}
\]

Now a capital market is introduced that allows both lending (S > 0) and borrowing (S < 0) at the market-determined real interest rate of r. The equilibrium r occurs when the savings of one family are equal to the dissavings of the other family. If 1 + r is lower than the return on investment in children, \((K_2 - x_2)/x_1'\), for some family, then it pays that family to borrow (and invest in children) indefinitely, thereby increasing future consumption indefinitely. But this cannot be an equilibrium. Thus, the equilibrium interest rate \(r^*\) cannot fall short of the rate of return on investment in children for any family:

\[
1 + r^* \geq \left(\frac{(K_2 - x_2)}{x_1'}\right) > \frac{(K_2 - x_2)}{x_1'}. \tag{29}
\]

Now, if the first inequality in equation (29) is strict, then both families have zero demand for children (because the rate of return on children is lower than the rate of interest). In this case, both families will want to save as this becomes the only means of securing positive second-period consumption, which is essential given the Cobb-Douglas specification of the utility function. But the capital market cannot be in equilibrium when both families save. Thus, at equilibrium,

\[
1 + r^* = \left(\frac{(K_2 - x_2)}{x_1'}\right) > \frac{(K_2 - x_2)}{x_1'}. \tag{30}
\]

In this case, family A will choose to have no children, \(n^* = 0\) because it is better for it to invest in the capital market. The first-period and second-period budget constraints, equations (15) and (16), now become:

\[
K_1 = \bar{c} + S, \tag{31}
\]

and

\[
\bar{c} = (1 + r)S. \tag{32}
\]
These two constraints can be consolidated into one present-value, lifetime budget constraint:

\[ K_1 = c + \bar{c}/(1 + r). \]  

(33)

Maximization of the utility function, equation (21), subject to the budget constraint, equation (33), yields the optimal levels of \( \bar{c}, \), \( \bar{c}^* \), and \( S \) for family A:

\[ \bar{c}^* A = \alpha K_1, \]  

(34)

\[ \bar{c}^* A = (1 - \alpha)K_1(1 + r), \]  

(35)

and

\[ S^* A = K_1 - \bar{c}^* A = (1 - \alpha)K_1. \]  

(36)

Family B is indifferent between investing in the capital market and investing in children (because \([K_2 - x_2]/x_1 = 1 + r^*\)). The consumption of this family is given by

\[ \bar{c}^* B = \alpha K_1, \]  

(37)

and

\[ \bar{c}^* B = (1 - \alpha)K_1(1 + r^*). \]  

(38)

For equilibrium in the capital market, family B must dissave, that is, family B must borrow in the capital market in order to invest in children (because family A has a positive \( S \)). Accordingly,

\[ S^* B = -S^* A = -(1 - \alpha)K_1. \]  

(39)

In order to find the number of children of family B, substitute equations (37) and (39) into the first-period budget constraint, equation (15), to obtain

\[ n^* B = (K_1 - c^* B - S^* B)/x_1^B = [2(1 - \alpha)K_1]/x_1^B. \]  

(40)

The aggregate number of children (\( N^* \)) in this case is found from equation (40) and the condition that \( n^* A = 0 \):

\[ N^* = n^* A + n^* B = [2(1 - \alpha)K_1]/x_1^B. \]  

(41)

Comparing \( N^* \) to \( N^{**} \) from equations (28) and (41), it can be seen that

\[ N^{**} = [2(1 - \alpha)K_1]/x_1^B = (1 - \alpha)K_1(1/x_1^B + 1/x_1^B) \]

\[ > (1 - \alpha)K_1(1/x_1^* + 1/x_1^*) = N^*. \]  

(42)
Thus, the introduction of a capital market increases, rather than decreases, the number of children, contrary to the old-age security hypothesis.\footnote{Readers familiar with international trade theory will recognize the result obtained in this example. Since operating on the capital market requires that a lender has to find a borrower (and vice versa), it involves trade between individuals. Thus the absence of a capital market corresponds to the familiar autarky equilibrium of trade theory, where the relative prices (that is, rates of return to investment) between autarkic individuals differ. Opening capital markets, that is, allowing lending and borrowing between individuals, brings about a common equilibrium rate of return. In standard trade models under usual assumptions, the equilibrium price after trade will lie in the interval between the two autarky prices. Compared to autarky, however, the output of one commodity will be lower in a trading equilibrium in the home country and higher in the foreign country, and the output of the second commodity is higher at home and lower abroad. But there is nothing to preclude the loss of output of either commodity in one country being more than offset by the gain in the output of the same commodity in the other country. Indeed, in the classic Ricardian world, after trade, England, specializing in linen, may produce more linen than she and Portugal together did in autarky. Thus, world output of linen may be higher than in autarky. Similarly, opening up capital markets may raise rather than lower the total number of children. We are indebted to T. N. Srinivasan for pointing out this parallel.}

\section*{Income and Substitution Effects With Endogenous Fertility}

So far it has been assumed that neither the number of children nor children's welfare entered the parents' utility function. If, however, parents do care about their children, it can again be shown that there is no presumption that the existence of a capital market will lead to a smaller demand for children than the absence of one.

Suppose, then, that the utility function is

\[ u = u(c, \hat{c}, x_1, x_2, n), \tag{43} \]

so that parents care about the number of their children, \( n \), and their children's welfare, which, in turn, depends on the children's consumption, \( x_1 \) and \( x_2 \). The parents now choose \( x_1 \) and \( x_2 \) as well as \( n \), \( \hat{c} \), and \( \hat{c} \).

In this case, the introduction of a capital market for transferring present to future consumption may plausibly increase the demand for children even in a partial equilibrium setting of the kind employed above, where the interest rate is exogenously given and there is no dissaving. This is because better access to capital markets increases welfare and thus may create a positive income effect on the desired number of children. This effect may dominate the negative substitution effect (as shown above) that better access to capital markets may have on the number of children.

In conclusion, better access to capital markets, by itself, need not imply theoretically a lower population growth rate. Empirical research focused on developing countries is needed on the old-age security hypothesis to establish what effect improvement in capital markets has on the population growth rate.
SOCIALLY OPTIMAL POPULATION SIZE: BEYOND THE PARETO PRINCIPLE

Criteria for a social optimum usually concern choices in which the number and identity of the individuals are given: in this case, although there are many difficulties of comparability, the criteria are otherwise unambiguous.

The classical utilitarian criterion is to maximize the sum of individual utilities:

\[ W^b(u^1, \ldots, u^n) = \sum_{h=1}^{n} u^h. \]

\( W^b \) can be called a Benthamite social welfare function. Since scaling all utilities up or down by a constant multiplicative factor does not affect any essential property of \( W \) if \( n \) is known, this criterion does not differ from the maximization of average or per capita utility:

\[ W^m(u^1, \ldots, u^n) = \frac{1}{n} \sum_{h=1}^{n} u^h. \]

\( W^m \) can be called a Millian social welfare function. But in a situation in which different choices produce different sizes of population, the two criteria can lead to different conclusions. For example, suppose that the question is whether to add an additional person to the existing population. If the utility of the additional person called into existence is positive but less than the average of the population in the status quo, then adding the person will produce a greater total utility but a smaller average.

The purpose here is not to decide what criterion should be used but rather to compare these two with each other and with the laissez-faire solution when fertility is endogenous. The analysis shows that the Benthamite social welfare function always leads to a larger population than the Millian criterion, and that the laissez-faire solution can yield a population either larger than the Benthamite or smaller than the Millian.

The analysis is made for a two-generation case, but the result can be extended to any finite number of generations and to an infinite number of generations, provided only that the infinite-generation case is restricted to stable population growth paths (so that the relationship between two consecutive generations is always the same).

Consider an economy with two generations, each consisting of just one type of consumer. In the first period, there is only one adult. He or she consumes (together with the children) a single private good (c\(^1\)). He or she also raises identical children who will grow up in the second period. He or she dies at the end of the first period and bequeaths b to each one of the children. The number of children (n) that are born in the first period is a decision variable of the parent living then. The number of persons living in the second period is n. Each one consumes a single private good (c\(^2\)). It is assumed that the parent cares about both the number and welfare of the children. Therefore, the children’s utilities are included in the parent’s utility function. In a reduced form, the parent’s utility can be written as

\[ u^1 = u^1[c^1, n, u^2(c^2)]. \]  

(44)
u₁ is concave in c₁ and u²; u² is increasing and concave in c²; both u₁ and u² are nonnegative (people enjoy positive happiness). u₁ is also increasing in c₁ and u², but it is not necessarily increasing in the number of children, n. Assume that the parent lives only one period and has a budget constraint of

\[ c₁ + nb = K; \quad c₁, n ≥ 0, \quad (45) \]

where K is the parent's initial endowment, which is nonrenewable and does not depreciate over time. This is like having an exhaustible resource capable of producing K units of consumption. The exact specification of supply is not important for this problem although for some issues it would be important to introduce production and capital accumulation.

Assume that children are born with no endowment. Thus the exhaustible resource has to suffice for the consumption of the current generation and all future generations. The children's per capita consumption is therefore equal to their per capita inheritance:

\[ c² = b. \quad (46) \]

Although the bequest, b, is not restricted to being nonnegative, it is immediately seen from equation (46) that it will never be negative. Thus institutional arrangements that do not allow b to be negative—parents cannot obligate their children to pay their debts—are superfluous here.

The constraints given by equations (45) and (46) can be consolidated into one budget constraint for the parent:

\[ c₁ + nc² = K. \quad (47) \]

A competitive or laissez-faire allocation (LFA) is obtained when the parent's utility function, equation (44), is maximized with respect to c₁, c², and n, subject to the budget constraint of equation (47). This allocation is denoted by (c₁L, c²L, nL).

In this model the Benthamite social welfare function is defined by:

\[ B(c₁, c², n) = u₁[c₁, n, u²(c²)] + nu²(c²). \quad (48) \]

As mentioned, it is assumed that there is diminishing marginal utility of c₁ and c², that is, u₁₁, u²₁₁ < 0, where the subscripts stand for partial derivatives. A Benthamite optimal allocation (BOA) is obtained by maximizing equation (48) with respect to c₁, c², and n, subject to the economy-wide budget constraint, equation (47). This allocation is denoted by (c₁B, c²B, nB).

The Millian social welfare function, namely the per capita utility, is

\[ M(c₁, c², n) = [u₁[c₁, n, u²(c²)] + nu²(c²)]/(1 + n) = [B(c₁, c², n)]/(1 + n). \quad (49) \]

The Millian optimal allocation (MOA) is obtained by maximizing equation (49) with respect to c₁, c², and n, subject to the resource constraint of equation (47). This allocation is denoted by (c₁M, c²M, nM).

It is important to emphasize that it is assumed that the parent's utility function represents his or her interest (for example, happiness from being a parent, guilt relief in providing for the children, and so on) rather than his or her moral (social) preferences.
(for example, believing that it would be wrong to have children and let them starve). This is why \(nu^2(c^2)\) are added to \(u^1[c^1, n, u^2(c^2)]\) when the Benthamite and Millian social welfare criteria are defined. In this way, children are seen as having more than just an instrumental role in society, that is, they are persons and individuals and not just the means or instruments by which parental welfare is affected.

One would expect that when the total happiness of society is maximized rather than the average happiness of its members, optimal population size will be larger. This is indeed the case: \(n^B > n^M\). To prove this, observe that both the BOA and the MOA satisfy the same resource constraint, equation (47). Since the Millian allocation maximizes \(M\) and since \(M = B/(1 + n)\), it follows that

\[
B(c^{1M}, c^{2M}, n^M)/(1 + n^M) \geq B(c^{1B}, c^{2B}, n^B)/(1 + n^B).
\]  

(50)

Since \((c^{1B}, c^{2B}, n^B)\) maximizes \(B\), it follows that

\[
B(c^{1B}, c^{2B}, n^B) \geq B(c^{1M}, c^{2M}, n^M).
\]  

(51)

Therefore,

\[
(1 + n^M)/(1 + n^B) \leq B(c^{1M}, c^{2M}, n^M)/B(c^{1B}, c^{2B}, n^B) \leq 1,
\]  

(52)

from which it follows that \(n^B \geq n^M\).

Since the Millian criterion calls for maximization of the average utility, intuition suggests that laissez-faire results in overpopulation. Although this may be true under some circumstances, it does not hold in general. Since the LFA satisfied the same resource constraint, as does the MOA, equation (47), it follows from the definition of the MOA that

\[
M(c^{1M}, c^{2M}, n^M) \geq M(c^{1L}, c^{2L}, n^L).
\]  

(53)

Since \(M = B/(1 + n)\), it is implied by equation (49) that

\[
B(c^{1M}, c^{2M}, n^M) \leq [(1 + n^M)/(1 + n^L)]B(c^{1L}, c^{2L}, n^L).
\]  

(54)

Since \(u^2 > 0\), it also follows that

\[
B(c^{1L}, c^{2L}, n^L) = u^1[c^{1L}, n^L, u^2(c^{2L})] + n^L u^2(c^{2L})
\geq u^1[c^{1L}, n^L, u^2(c^{2L})] \geq u^1[c^{1M}, n^M, u^2(c^{2M})]
\]  

(55)

because \((c^{1L}, c^{2L}, n^L)\) maximizes \(u^1\), subject to the overall resource constraint, equation (47). Thus it can be concluded from equations (54) and (55) that

\[
B(c^{1M}, c^{2M}, n^M) \geq [(1 + n^M)/(1 + n^L)]u^1[c^{1M}, n^M, u^2(2^{2M})],
\]

so that

---

\(^{34}\) This method of proof was suggested by T. N. Srinivasan.
\[ (1 + n^M)/(1 + n^L) \leq B(c^{1M}, c^{2M}, n^M)/u^1[c^{1M}, n^M, u^2(c^{2M})] \]
\[ = \{u^1[c^{1M}, n^M, u^2(c^{2M})] + n^M u^2(c^{2M})\}/u^1[c^{1M}, n^M, u^2(c^{2M})] \]
\[ = 1 + \left\{ n^M u^2(c^{2M})/u^1[c^{1M}, n^M, u^2(c^{2M})] \right\}. \quad (56) \]

Since the extreme right-hand side of equation (56) is strictly greater than 1, it is impossible to say anything about the ratio on the extreme left-hand side; in particular, it cannot be concluded that \( n^L > n^M \).

Since the Benthamite criterion calls for a maximization of the total utility of parents and children while the competitive allocation maximizes the parent’s utility only, intuition suggests that laissez-faire leads to a smaller population than the Benthamite optimum. However, this is not necessarily true: when \( nu^2(c^2) \) is added to the parent’s utility, as suggested by the Benthamite criterion, increasing the product \( nu^2(c^2) \) is indeed desirable; but it does not follow that both \( n \) and \( c^2 \) need be increased.

To see this, observe that it follows from the definition of the LFA and the BOA that
\[ u^1[c^{1L}, n^L, u^2(c^{2L})] \geq u^1[c^{1L}, n^B, u^2(c^{2B})], \]
and
\[ u^1[c^{1B}, n^B, u^2(c^{2B})] + n^B u^2(c^{2B}) \geq u^1[c^{1L}, n^L, u^2(c^{2L})] + n^L u^2(c^{2L}). \]
Hence,
\[ n^B u^2(c^{2B}) \geq n^L u^2(c^{2L}). \]

Thus, indeed the total utility from children (\( nu^2 \)) must be larger at the BOA than at the LFA. But it does not follow that \( n^B > n^L \).

The assumption that fertility is endogenous allows consideration of noncoercive policies aimed at moving the economy from the LFA to either the BOA or the MOA by changing the incentives (prices) that parents face. Notice that the need here for government action, unlike in the externality case, is not warranted because of a market failure. The LFA is indeed efficient. It is located on the Pareto frontier of Figure 2. Government action is needed only to move to the socially optimal allocation, which is another point on this frontier.

All possible direct and indirect taxes and subsidies are considered candidates for the optimal policy. Notice that children themselves are a “commodity” and may be subject to a tax or a subsidy. As explained in Chapter 5, a head tax is not a lump-sum nondistortionary tax as in the traditional economic literature with exogenous population. Here such a head tax affects fertility decisions on the margin.

Among the set of possible direct and indirect taxes and subsidies to achieve a social optimum, subsidies for future consumption and child allowances (positive or negative to encourage or discourage having children) are necessary. It can be shown that a subsidy for future consumption (\( c^2 \)) is warranted under both the Benthamite and the Millian criteria; a positive child allowance is necessary under the Benthamite criterion, but the child allowance needed under the Millian criterion may be positive, zero, or negative.

Although the remedies that are needed here are not necessitated by externalities, a similar apparatus can nevertheless be employed to derive them. The difference between the social welfare function and the parent’s utility implies that marginal social benefits (as derived from the social welfare function) will, in general, be different from

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Figure 9—Optimal numbers of children and their consumption with a Benthamite social welfare function

Marginal private benefits (as derived from the parent's utility). This is why techniques can be used here that are similar to those used in the externality case. Consider first the BOA. It is obtained by maximizing

$$u^1[c^1, n, u^2(c^2)] + nu^2(c^2),$$

subject to the resource constraint:

$$k - c^1 - nc^2 = 0$$

(see equations [47] and [48] above).

The marginal cost of $c^2$ is the same for the parent and society. It is derived from the budget constraint, equation (47), as the number of children. The $MSC = MPC$ curve is drawn in Figure 9. However, the benefits differ. For the parent, the marginal benefit (MPB) is the marginal utility that the parent derives from $c^2$, which is $u^1 u^2 / u^1$. Expressed in units of the all-purpose good, $MPB = u^1 u^2 / u^2$. However, society extracts utility from children not only via the parent's ability, but also directly via the term $nu^2$. Hence $MSB = MPB + nu^2 / u^1$. Thus the Benthamite optimum consumption per child is at $c^{2B}$, where $MSB = MSC$. To support this amount of consumption, the parent's cost of $c^2$ should be lowered by an appropriate subsidy, so that the $MPC$ curve will intersect the $MPB$ curve at $c^{2B}$. Similarly, the marginal social benefit of children exceeds the marginal private benefit of children because of the term $nu^2$, which is added to
Table 1—Optimal policies under the Benthamite and Millian optima

<table>
<thead>
<tr>
<th>Policy</th>
<th>Benthamite Optimum Allocation</th>
<th>Millian Optimum Allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subsidy to children's</td>
<td></td>
<td></td>
</tr>
<tr>
<td>consumption</td>
<td>( u_1^2 / u_1^1 &gt; 0 )</td>
<td>( u_2^2 / u_1^1 &gt; 0 )</td>
</tr>
<tr>
<td>Child allowance</td>
<td>( u_2^2 / u_1^1 - c_2 u_1^2 / u_1^1 &gt; 0 )</td>
<td>( u_2^2 / u_1^1 - c_2 u_1^2 / u_1^1 - u_1^1 + n M u_1^2 / (1 + n M) u_1^1 )</td>
</tr>
</tbody>
</table>

Note: The derivation of these formulae is given in the Appendix.

the parent's utility, \( u_1 \). Hence, a child allowance is needed in order to support the BOA. In a similar fashion, one can derive the appropriate measures needed to support the MOA. The exact formulae for the policies needed to support the BOA and MOA are given in Table 1. The derivation of these formulae is in the Appendix.
DIRECTIONS FOR FURTHER RESEARCH

This report has explored only the simplest feature of endogenous fertility in a purely static context.\textsuperscript{35} As a postscript to this investigation, several promising directions of further investigation are outlined. These are, first, the general theory of tax and transfer policy that considers family size and composition. Second, alternative provision and finance of public goods. Third, intragenerational income distribution with household production and endogenous fertility. Fourth, heterogeneous family preferences. Fifth, uncertainty with respect to child quality. And lastly, the implications of endogenous fertility for population dynamics.

Tax and Transfer Policies

An extension of the basic model to emphasize the population aspects (quantity and quality) of tax policies can be made as follows. One wants to devise an income tax schedule that depends on family income, family size and age composition, and quality-improving expenditures. Consider a stylized model of overlapping generations in which each family lives for three periods: a representative individual is born in period 1, during which he or she has no income of his or her own; the individual works (earning \(l\), which depends on the individual's human capital stock) and bears children in period 2; the individual retires in period 3. In period 1, all decisions with regard to consumption, education, and so forth are made by the individual's parents. In period 2, the individual decides the number of children to have \((n)\) and how much to invest in them \((z)\), how much the family shall consume \((c_2)\), and how much to save for retirement \((s)\). In period 3, the individual decides how much accumulated savings plus any transfer income (social security benefits) to consume \((c_3)\) and how much to bequeath to his or her children \((b)\). A general tax-transfer function can be represented by a vector function:

\[
T = (T_2, T_3) = \{T_2(l, n, z, s), T_3[s(1 + r), b]\},
\]

where \(r\) is the interest rate. This function incorporates various tax-transfer instruments, such as income or consumption taxes, social security taxes, inheritance taxes, social security benefits, child allowances or deductions, and educational subsidies or deductions. For example, an income tax prevails when \(T_2\) depends on \(l\) and does not depend on \(s\); a consumption tax is obtained when \(T_2\) depends on \(1 - s\). Child deductions are incorporated by making \(T_2\) depend on \(1 - n\beta\), where \(\beta\) is the per capita child exemption. In the third period, \(T_3\) represents the net tax payments, which may be negative if social security payments outweigh tax payments. An inheritance tax is represented by having \(T_3\) depend on \(b\).

\textsuperscript{35} This chapter is adapted from Household and Economy: Welfare Economics of Endogenous Fertility, by Marc Nerlove, Assaf Razin, and Efrain Sadka. Copyright © 1987 by Academic Press, Inc. Reprinted by permission of the publisher.
Extending the model—by evaluating a number of possible specific forms of this general tax function, assessing the implications of each tax policy on family behavior, and deriving the optimal tax policy—puts the treatment of family size and composition in the theory of taxation on a firm microeconomic basis.

Public Goods and Dynasty Taxes

In Chapter 5 it was assumed that the government provides the public goods in each period and finances them by a lump-sum tax (T) that is imposed on the dynasty as a whole. (This approach is necessary because a head tax is not a lump-sum tax in this model, as the number of children is endogenous.) Under this assumption, it was shown that the existence of public goods does not lead to market failure. On the other hand, a head tax on individual members of each generation is obviously distortionary when fertility is endogenous and will, in general, lead to market failure. If a tax on each dynasty is ruled out, one may consider two alternatives.

First, the public good may be financed by a tax on land rent (pure economic profit). In this case, there will be no market failure, provided that there is enough rent on land to finance the public good. Indeed, the theory of local public finance (the “Henry George Rule”) suggests that if the quantity of each public good is set optimally at each point in time, then a 100 percent tax on land rent will be just sufficient to finance provision of the good. This theory, however, has not been developed for the case of endogenous fertility and the “Rule” is not obviously true. Can a first-best solution to the problem of providing public goods be achieved if a tax on each dynasty is ruled out?

Second, with no dynasty taxes and no full optimal provision of public goods through land taxes, one can consider second-best solutions, among them those achieved through head taxes on individual members of each generation, income taxes, interest taxes, taxes on labor income, inheritance taxes, child allowances and taxes, and so forth. In doing so, one could develop a theory of the second-best optimal provision of public goods, population size, and tax financing.

As discussed in Chapter 5, if marriages are allowed between dynasties, then a market failure can arise. In this case, children who marry children from another dynasty reduce the average tax burden on each original member of the other dynasty, and vice versa. Thus there is an external economy to the number of children that is not internalized by the heads of dynasties. A similar kind of externality associated with marriage also applies to transfers between generations: the transfer that parents make to their children also benefits the parents of the spouses of the children. This particular issue was considered in Chapter 6, but the same framework can easily be applied to the external economy that comes from marriage between dynasties in the presence of public goods.

Intragenerational Income Distribution With Household Production and Endogenous Fertility

Chapter 8 deals with some implications endogenous fertility has for the effects that alternative policies on income taxation and family allowances have on the distribution of welfare between generations. In further research it would be important to take account of the determinants of fertility beyond merely including the number and quality of children in a reduced form of the utility function and the budget constraints of the family. In particular, it may prove useful to use a household production function to
determine the allocation of time and resources within a family among market activities, child rearing, and other activities. Such analysis has important implications for the determination of tax policies concerning the treatment of the number of children, for first and second wage earners in families, for the age of wage earners and children, and so forth. This kind of analysis may be extended in the context of the Becker-Lancaster theory of time allocation and household production.

**Heterogeneous Family Preferences**

In all of the work to date on population size and bequests, the assumption that a single utility function represents family preferences has generally been maintained. But it may be far more appropriate to consider different objectives for husbands and wives, which may generate intrafamily conflicts and necessitate a contractual or other theory of household behavior.

**Uncertainty With Respect to Child Quality**

Chapter 5 dealt with some issues raised by heterogeneous child quality. Given constraints on the ability of parents to enforce transfers among their offspring, laissez-faire leads to a genuine market failure: it was found that a tax on earned income can improve the distribution of welfare among members of the current generation. If child quality, in the sense of being able to absorb investment in human capital productively, is uncertain ex ante, new difficulties arise. If the uncertainty pertains only to individual families but not collectively, it is possible to design a social insurance scheme that will permit an optimal solution for the number of children, investment in them, and bequests of nonhuman capital. How should such a first-best insurance policy be characterized? If for reasons of moral hazard or other problems, insurance is ruled out, what are the second-best alternatives, such as taxes on earned income?

**Implications of Endogenous Fertility for Population Dynamics**

The most pressing population problems in the developing world today are not those stemming from the size of population per se but from the path that population follows over time. The analysis in this report of the welfare implications of endogenous fertility is restricted to an equilibrium framework. Much of the debate over population policy, however, centers on the dynamic process of demographic change and what are essentially disequilibrium phenomena. The implications of endogenous fertility for the changing costs of, and benefits from, children are far-reaching. An explanation of the so-called “demographic transition” in terms of individual responses to changing costs and opportunities has yet to be given.

Such extensions and deepening of the analysis lie in the future. The present work makes a contribution in showing that a fuller integration of the empirical and theoretical insights of the microtheory of endogenous fertility into a general equilibrium and welfare-theoretic analysis of population growth and the relations among generations has a high payoff.
APPENDIX: DERIVATION OF FORMULAE FOR POLICIES NEEDED TO SUPPORT THE BENTHAMITE AND MILLIAN OPTIMA

The BOA is obtained by maximizing

\[ u^1[c^1, n, u^2(c^2)] + nu^2(c^2), \]

subject to the budget constraint

\[ K - c^1 - nc^2 = 0. \]

Letting \( \lambda \geq 0 \) be the Lagrange multiplier, the following first-order conditions for an interior solution may be derived:

\[ u^1 = \lambda, \quad u^1 + u^2 = \lambda c^2, \quad \text{and} \]
\[ u^1 u^2 + nu^2 = \lambda n. \]

Dividing equations (58) and (59) by equation (57),

\[ (u^1 + u^2)/u^1 = c^2, \quad \text{and} \]
\[ (u^1 u^2 + nu^2)/u^1 = n. \]

Equation (60) asserts that the social marginal rate of substitution of \( c^1 \) for \( n \) (the willingness of society to give up a parent’s consumption for an additional child), which is \((u^1 + u^2)/u^1\), must be equated to the social “cost” of an additional child, which is equal to its consumption, \( c^2 \). Similarly, equation (61) asserts that the social marginal rate of substitution of \( c^1 \) for \( c^2 \) must be equated to the social “cost” of a unit of the child’s consumption, which is \( n \), since every one of the \( n \) children consumes this unit.

In order to achieve the BOA allocation (via the market mechanism), it may be possible for the government to subsidize \( c^2 \) at the rate of \( \alpha \), to give child allowances (possibly negative) of \( \beta \) per child, and to balance its budget by a lump-sum tax (possibly negative) in the amount \( T \). In this case, the parent’s budget constraint becomes

\[ c^1 + nc^2(1 - \alpha) = K + \beta n - T. \]

Given this budget constraint, the parent maximizes

\[ u^1[c^1, n, u^2(c^2)] \]

by choosing \( c^1 \), \( n \), and \( c^2 \). Letting \( \theta \geq 0 \) be the Lagrange multiplier for this problem, the following first-order conditions for an interior solution are

\[ u^1 = \theta, \quad \text{and} \]

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\[ u_2 = -\theta \beta + \theta c^2 (1 - \alpha), \quad \text{and} \quad (64) \]
\[ u_1 u_2 = \theta n (1 - \alpha). \quad (65) \]

Dividing equations (64) and (65) by equation (63),

\[ \frac{u_2}{u_1} = c^2 (1 - \alpha) - \beta, \quad \text{and} \quad (66) \]
\[ \frac{u_1 u_2}{u_1} = n (1 - \alpha). \quad (67) \]

Equation (66) states that the marginal rate of substitution of \( c^1 \) for \( n \) (that is, a parent's willingness to give up his or her own consumption for an additional child) must be equated to the "price" of a child as perceived by the parent from the budget constraint, equation (62). The "price" consists of two components: the cost of providing the child with \( c^2 \) units of consumption, which is only \( c^2 (1 - \alpha) \) because of the subsidy \( \alpha \), and the tax on children, which is \(-\beta\). Equation (67) states that the marginal rate of substitution of \( c^1 \) for \( c^2 \) must be equated to the "price" of \( c^2 \), which is the number of children, times \( 1 - \alpha \).

If it is possible to achieve a BOA in this way, the optimal values of \( \alpha \) and \( \beta \) can be found by comparing the first-order conditions for the BOA (namely, equations [60] and [61]) with those of the individual parent's optimization problem (equations [66] and [67]). First, compare equation (61) with equation (67) to conclude that

\[ n (1 - \alpha) = n [1 - (u_2/u_1)], \]

so that the optimal subsidy to children's consumption under the Benthamite criterion is

\[ \alpha^B = \frac{u_1 [c^{1B}, n^B, u^2 (c^{2B})]}{[c^{1B}, n^B, u^2 (c^{2B})]]. \quad (68) \]

Next, compare equation (60) with equation (66) to conclude that

\[ c^2 (u_2/u_1) = c^2 (1 - \alpha) - \beta, \]

so that the optimal child allowance under the Benthamite criterion is

\[ \beta^B = \frac{u^2 (c^{2B})}{u_1 [c^{1B}, n^B, u^2 (c^{2B})]} - \alpha^B c^{2B}. \quad (69) \]

The interpretation of the formulae for \( \alpha \) and \( \beta \) is straightforward. Since the term \( nu^2 (c^2) \) of the Benthamite criterion, equation (48), is ignored by the parent objective, \( c^2 \) generates a difference between private and social evaluations; hence it ought to be subsidized in order to achieve the BOA. The optimal size of this subsidy has to be determined according to what the parent ignores (at the margin). When the parent considers increasing \( c^2 \), he or she ignores the social benefit \( nu^2_1 \) at the margin. This benefit is measured in utility units. Its equivalent in terms of the numeraire consumption good is \( nu^2_2/u_1 \). From the parent's budget constraint, equation (62), it can be seen that if \( c^2 \) is subsidized at the rate \( \alpha \), then each unit of \( c^2 \) receives a subsidy of \( n \alpha \). Thus the subsidy ought to be set so that \( n \alpha = nu^2_2/u_1 \), which explains the value of the optimal \( \alpha \) in equation (68).

For the same reason, \( n \) ought to be subsidized by \( u^2_2/u_1 \), so that the price of \( n \) for the parent will be \( c^2 - (u^2_2/u_1) \). Since by the parent's budget constraint, equation (62),
the price of \( n \) is \( c^2(1 - \alpha) - \beta, \) \( c^2 - (u^2/u^1) \) must equal \( c^2(1 - \alpha) - \beta. \) Thus, it follows that \( \beta^B = (u^2/u^1) - \alpha^B c^2, \) as in equation (69).

Note that \( \alpha^B > 0. \) To find the sign of \( \beta^B, \) observe that

\[
\beta^B = (u^2/u^1) - \alpha^B c^2 = (u^2 - c^2 u^1)/u^1,
\]

by substituting equation (68) into equation (69). Since \( u^2 \) is concave, it follows that

\[
u^2(c^2) - u^2(0) \geq u^2_1(c^2)(c^2 - 0)\]

Since \( u^2 \) is assumed to be nonnegative, it follows that

\[
u^2_1(c^2) \geq c^2 u^2_1(c^2),
\]

so that \( \beta^B > 0: \) the optimal child allowance under the Benthamite criterion must be positive.

Fixed \( \alpha \) and \( \beta \) may not in fact lead to the BOA because the parent’s optimization problem is not convex; therefore the second-order conditions may not hold. If they do not hold with fixed \( \alpha \) and \( \beta, \) it is possible to achieve a BOA with nonlinear taxes, that is, with instruments \( \alpha \) and \( \beta, \) which are functions of \( c^1, c^2, \) and \( n. \) In other words, the second-order conditions can always be satisfied by functions \( \alpha(\cdot) \) and \( \beta(\cdot). \) The values of \( \alpha(\cdot) \) and \( \beta(\cdot) \) at the optimum will be exactly \( \alpha^B \) and \( \beta^B \) as given in equations (68) and (69), that is,

\[
\alpha^B = \alpha(c^1B, n^B, c^2B),
\]

and

\[
\beta^B = \beta(c^1B, n^B, c^2B).
\]

Exactly the same kind of analysis may be carried through for the Millian social welfare function. In this case, one finds that the subsidy to children’s consumption, namely \( \alpha^M, \) is positive, as in the Benthamite case. But the sign of the optimal child allowance \( \beta^M \) is ambiguous. The reason for this ambiguity can be seen by comparing the Millian objective, which is \( (u^1 + nu^2)/(1 + n), \) with the parent’s objective, which is just \( u^1. \) On the one hand, the Millian objective adds \( nu^2 \) to the parent’s objective. In this way \( n \) increases MSB above MPB. But, on the other hand, \( u^1 + nu^2 \) is divided by \( 1 + n. \) In this way, \( n \) lowers MSB. Thus, one cannot determine a priori whether \( n \) should be taxed or subsidized.
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